Chapter 2

2.1-1 Let us denote the signal in question by $g(t)$ and its energy by E_g. For parts (a) and (b)

$$E_g = \int_0^{2\pi} \sin^2 t\,dt = \frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t\,dt = \pi + 0 = \pi$$

(c) $$E_g = \int_{2\pi}^{4\pi} \sin^2 t\,dt = \frac{1}{2}\int_{2\pi}^{4\pi} dt - \frac{1}{2}\int_{2\pi}^{4\pi} \cos 2t\,dt = \pi + 0 = \pi$$

(d) $$E_g = \int_0^{2\pi} (2\sin t)^2\,dt = 4\left[\frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t\,dt\right] = 4[\pi + 0] = 4\pi$$

Sign change and time shift do not affect the signal energy. Doubling the signal quadruples its energy. In the same way we can show that the energy of $kg(t)$ is $k^2 E_g$.

2.1-2 (a) $E_x = \int_0^2 (1)^2 dt = 2.$ $\quad E_y = \int_0^1 (1)^2 dt + \int_1^2 (-1)^2 dt = 2$

$$E_{x+y} = \int_0^1 (2)^2 dt = 4.\quad E_{x-y} = \int_1^2 (2)^2 dt = 4$$

Therefore $E_{x\pm y} = E_x + E_y$.

(b) $$E_x = \int_0^{\pi} (1)^2 dt + \int_{\pi}^{2\pi} (-1)^2 dt = 2\pi. \qquad E_y = \int_0^{\pi/2} (1)^2 dt + \int_{\pi/2}^{\pi} (-1)^2 dt + \int_{\pi}^{3\pi/2} (1)^2 dt + \int_{3\pi/2}^{2\pi} (-1)^2 dt = 2\pi$$

$$E_{x+y} = \int_0^{\pi/2} (2)^2 dt + \int_{\pi/2}^{3\pi/2} (0)^2 dt + \int_{3\pi/2}^{2\pi} (-1)^2 dt = 4\pi$$

Similarly, we can show that $E_{x-y} = 4\pi$ Therefore $E_{x\pm y} = E_x + E_y$. We are tempted to conclude that $E_{x\pm y} = E_x + E_y$ in general. Let us see.

(c) $$E_x = \int_0^{\pi/4} (1)^2 dt + \int_{\pi/4}^{\pi} (-1)^2 dt = \pi \qquad E_y = \int_0^{\pi} (1)^2 dt = \pi$$

$$E_{x+y} = \int_0^{\pi/4} (2)^2 dt + \int_{\pi/4}^{\pi} (0)^2 dt = \pi \qquad E_{x-y} = \int_0^{\pi/4} (0)^2 dt + \int_{\pi/4}^{\pi} (-2)^2 dt = 3\pi$$

Therefore, in general $E_{x\pm y} \neq E_x + E_y$

2.1-3

$$P_g = \frac{1}{T_0}\int_0^{T_0} C^2 \cos^2(\omega_0 t + \theta)\,dt = \frac{C^2}{2T_0}\int_0^{T_0} [1 + \cos(2\omega_0 t + 2\theta)]\,dt$$

$$= \frac{C^2}{2T_0}\left[\int_0^{T_0} dt + \int_0^{T_0} \cos(2\omega_0 t + 2\theta)\,dt\right] = \frac{C^2}{2T_0}[T_0 + 0] = \frac{C^2}{2}$$

2.1-4 This problem is identical to Example 2.2b, except that $\omega_1 \neq \omega_2$. In this case, the third integral in P_g (see p. 19 is not zero. This integral is given by

$$I_3 = \lim_{T\to\infty} \frac{2C_1C_2}{T}\int_{-T/2}^{T/2} \cos(\omega_1 t + \theta_1)\cos(\omega_1 t + \theta_2)\,dt$$

$$= \lim_{T\to\infty} \frac{C_1C_2}{T}\left[\int_{-T/2}^{T/2} \cos(\theta_1 - \theta_2)\,dt + \int_{-T/2}^{T/2} \cos(2\omega_1 t + \theta_1 + \theta_2)\,dt\right]$$

$$= \lim_{T\to\infty} \frac{C_1C_2}{T}[T\cos(\theta_1 - \theta_2)] + 0 = C_1C_2\cos(\theta_1 - \theta_2)$$

Therefore

$$P_y = \frac{C_1^2}{2} + \frac{C_2^2}{2} + C_1 C_2 \cos(\theta_1 - \theta_2)$$

2.1-5

$$P_g = \frac{1}{4}\int_{-2}^{2}(t^3)^2 dt = 64/7 \qquad \textbf{(a) } P_{-g} = \frac{1}{4}\int_{-2}^{2}(-t^3)^2 dt = 64/7$$

$$\textbf{(b) } P_{2g} = \frac{1}{4}\int_{-2}^{2}(2t^3)^2 dt = 4(64/7) = 256/7 \qquad \textbf{(c) } P_{cg} = \frac{1}{4}\int_{-2}^{2}(ct^3)^2 dt = 64c^2/7$$

Sign change of a signal does not affect its power. Multiplication of a signal by a constant c increases the power by a factor c^2.

2.1-6

$$\textbf{(a)} \qquad P_g = \frac{1}{\pi}\int_0^{\pi}(e^{-t/2})^2 dt = \frac{1}{\pi}\int_0^{\pi} e^{-t} dt = \frac{1}{\pi}[1 - e^{-\pi}]$$

$$\textbf{(b)} \qquad P_g = \frac{1}{2\pi}\int_{-\pi}^{\pi} w^2(t)\, dt = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} dt = 0.5$$

$$\textbf{(c)} \qquad P_g = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} w_0^2(t)\, dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} dt = 1$$

$$\textbf{(d)} \qquad P_g = \frac{1}{4}\int_{-2}^{2}(\pm 1)^2 dt = 1$$

$$\textbf{(e)} \qquad P_g = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{t}{2\pi}\right)^2 dt = \frac{1}{3}$$

2.1-7

$$P_g = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} g(t)g^*(t)\, dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=m}^{n}\sum_{r=m}^{n} D_k D^*_r e^{j(\omega_k - \omega_r)t}\, dt$$

The integrals of the cross-product terms (when $k \neq r$) are finite because the integrands are periodic signals (made up of sinusoids). These terms, when divided by $T \to \infty$, yield zero. The remaining terms ($k = r$) yield

$$P_g = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=m}^{n}|D_k|^2 dt = \sum_{k=m}^{n}|D_k|^2$$

2.1-8 **(a)** Power of a sinusoid of amplitude C is $C^2/2$ [Eq. (2.6a)] regardless of its frequency ($\omega \neq 0$) and phase. Therefore, in this case $P = (10)^2/2 = 50$.
(b) Power of a sum of sinusoids is equal to the sum of the powers of the sinusoids [Eq. (2.6b)]. Therefore, in this case $P = \frac{(10)^2}{2} + \frac{(16)^2}{2} = 178$.
(c) $(10 + 2\sin 3t)\cos 10t = 10\cos 10t + \sin 13t - \sin 3t$. Hence from Eq. (2.6b) $P = \frac{(10)^2}{2} + \frac{1}{2} + \frac{1}{2} = 51$.
(d) $10\cos 5t\cos 10t = 5(\cos 5t + \cos 15t$. Hence from Eq. (2.6b) $P = \frac{(5)^2}{2} + \frac{(5)^2}{2} = 25$.
(e) $10\sin 5t\cos 10t = 5(\sin 15t - \sin 5t$. Hence from Eq. (2.6b) $P = \frac{(5)^2}{2} + \frac{(-5)^2}{2} = 25$.
(f) $e^{j\alpha t}\cos\omega_0 t = \frac{1}{2}\left[e^{j(\alpha+\omega_0)t} + e^{j(\alpha-\omega_0)t}\right]$. Using the result in Prob. 2.1-7, we obtain $P = (1/4) + (1/4) = 1/2$.

2.2-1 For a real a

$$E_g = \int_{-\infty}^{\infty}(e^{-at})^2 dt = \int_{-\infty}^{\infty} e^{-2at}\, dt = \infty$$

$$P_g = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}(e^{-at})^2 dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} e^{-2at}\, dt = \infty$$

For imaginary a, let $a = jr$. Then

$$P_g = l1imT\to\infty\frac{1}{T}\int_{-T/2}^{T/2}(e^{jrt})(e^{-jrt})\, dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} dt = 1$$

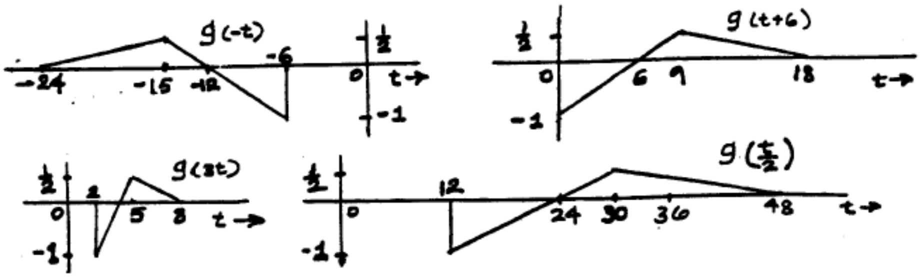

Fig. S2.3-2

Clearly, if a is real, e^{-at} is neither energy not power signal. However, if a is imaginary, it is a power signal with power 1.

2.3-1

$$g_2(t) = g(t-1) + g_1(t-1), \quad g_3(t) = g(t-1) + g_1(t+1), \quad g_4(t) = g(t-0.5) + g_1(t+0.5)$$

The signal $g_5(t)$ can be obtained by (i) delaying $g(t)$ by 1 second (replace t with $t-1$), (ii) then time-expanding by a factor 2 (replace t with $t/2$), (iii) then multiply with 1.5. Thus $g_5(t) = 1.5g(\frac{t}{2} - 1)$.

2.3-2 All the signals are shown in Fig. S2.3-2.

2.3-3 All the signals are shown in Fig. S2.3-3

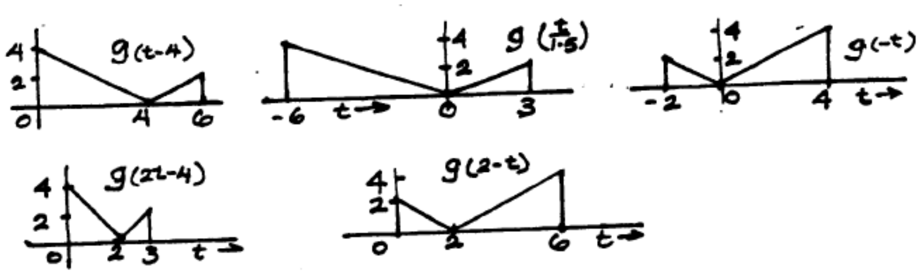

Fig. S2.3-3